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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2702

AN APPROXIMATE METHOD OF DETERMINING THE SUBSONIC FLOW IN AN ARBITRARY STREAM FILAMENT OF REVOLUTION CUT BY ARBITRARY TURBOMACHINE BLADES

By Chung-Hua Wu, Curtis A. Brown, and Vasily D. Prian

SUMMARY

A method is presented to obtain a relatively quick approximate determination of the detailed subsonic flow of a nonviscous fluid past arbitrary turbomachine blades. The governing equations are formulated as though the flow were constrained to stream filaments of revolution (the thicknesses of which vary as they pass through the machine). Attention is first fixed on a certain streamline lying between the blades and in such a filament. The shape of this streamline and the specific mass flow (product of density and velocity) along it could be estimated, for example, from the shape and the width of the channel formed by two adjacent blades and from the inlet conditions. From these starting values, the governing equations yield separate values of density and velocity components and their derivatives in the circumferential direction. These values are incorporated in a Taylor's series to provide the analytical continuation of the flow quantities from the chosen streamline to the blade surfaces. Considerations of integrated mass flow then show how the assumed shape of and specific mass flow along the chosen streamline must be adjusted to provide a starting point in a second cycle of computation.

The method is illustrated with examples of compressible flow in a turbine cascade and in a centrifugal compressor. For these high-solidity blades, three terms of the Taylor's series are found to give sufficient accuracy. Sufficient convergence is obtained in the turbine cascade after two cycles of computation (started with an available incompressible mean streamline) and in the centrifugal compressor after four cycles of computation (without any aid of available information). The detailed flow variation obtained compares very well with an available numerical solution and experimental data for the turbine cascade and with detailed experimental measurements for the centrifugal compressor. Because each cycle of computation for compressible flow takes only 16 hours, successive improvement of the solution is practical.

INTRODUCTION

A basic aerodynamic problem of gas-turbine engines is the flow of air or gas through the compressor or turbine bladings. When the distance between hub and casing is relatively short, the air or gas may be assumed, for an approximate solution, to flow along surfaces of revolution which are usually noncylindrical (figs. 1 and 2). The equations governing the flow of a nonviscous compressible fluid along such surfaces have been formulated recently by the use of a set of orthogonal coordinates l and φ (meridional and angular, respectively) on the surface (figs. 1 and 2), and general methods of solution for both subsonic and supersonic flow are given (references 1 and 2). In the case of supersonic flow, solution by the use of the method of characteristics is satisfactory. But in the case of subsonic flow, the general numerical method suggested takes a long time to accomplish if a high-speed large-scale digital computing machine is unavailable. In such a case, a much quicker approximate method of solution which gives sufficient accuracy is desirable. 2384

When the distance between hub and casing of the compressor or turbine is relatively large, the two-dimensional solution of the flow along the surfaces of revolution cut by the blade sections becomes inadequate. A general theory of three-dimensional flow is given in reference 3, in which the complete three-dimensional flow in a turbomachine is obtained by a suitable combination of two-dimensional flows along two kinds of stream surface extending, respectively, from hub to casing and from blade to blade (fig. 3). In the first stage of such calculations, it is desirable to have some general approximate knowledge of the detailed flow variation from blade to blade obtainable by relatively quick approximate methods by assuming that the gas flows along surfaces of revolution.

For these purposes, a quick approximate method of solution of the detailed subsonic flow of a nonviscous fluid past arbitrary turbomachine blade sections along an arbitrary surface of revolution has been developed at the NACA Lewis laboratory. The method is essentially an extension of the general two-dimensional blade design method reported in reference 4 and is particularly useful for high-solidity blades, such as those encountered in axial-flow turbines and radial- or mixed-flow compressors and turbines. The method is presented and is then illustrated by two examples, namely, compressible flow past a turbine cascade and compressible flow along a curved stream filament of revolution (at the mean blade height) in a centrifugal compressor. The velocity and pressure distributions along the blade surfaces as well as across the channel are compared with available numerical solutions or experimental measurements or both.

τ normal thickness of stream filament of revolution

ψ stream function

ω angular velocity of blade

Subscripts:

c on chosen streamline

i at inlet

l, φ meridional and circumferential components, respectively

m on mean streamline

p pressure surface of blade

s suction surface of blade

t total or stagnation state

y y-component

z z-component

Superscript:

* dimensionless value obtained by dividing by inlet value

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METHOD

Equations Governing Flow on Arbitrary Surface of Revolution

When the gas is assumed to be nonviscous and to flow steadily along surfaces of revolution in turbomachines, a relatively simple description formulated in reference 1 of the governing equations for the gas flow is applicable. For the gas flowing in an arbitrary stream filament of revolution defined by the meridional coordinate l and the angular coordinate φ of the mean surface of revolution and by the varying normal thickness τ of the stream filament (fig. 1), the continuity equation takes the following form:

$$\frac{\partial(\tau\rho W_l r)}{\partial l} + \frac{\partial(\tau\rho W_\varphi)}{\partial \varphi} = 0 \qquad (1)$$